A Probability Criterion for Acceptable Soil Erosion

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ABSTRACT

THE soil erosion process is presently considered acceptable whenever the predicted mean of the soil erosion distribution is equal to or less than the soil loss tolerance. Another criterion is proposed, which limits the soil erosion to a specified range with an acceptable degree of risk. The rationale, required assumptions, and methods are discussed for determining this criterion, which is a function only of soil productivity.

Because of this difficulty in simulation soil erosion, for the time implied by this criterion, a method is suggested for determining a short term erosion criteria.

INTRODUCTION

The present standard for judging the acceptability of predicted soil erosion is the soil loss tolerance, T (McCormack et al., 1982). The numerical values of T and the various methods for its estimation have been the subject of many papers and a few symposia. The proceedings of the most recent symposium (Schmidt et al., 1982) adequately covers the subject matter and includes a wealth of citations. As noted in the symposium proceedings preface, T (or the T-value) has been used as a conservation planning tool in conjunction with the Universal Soil Loss Equation (Wischmeier and Smith, 1978). When predicting soil loss due to both wind and water erosion, the Wind Erosion Equation (Woodruff and Siddoway, 1965; Skidmore and Woodruff, 1968) must also be used with a modification of the criteria, to

$$T \geqslant A + E_c . \qquad [1]$$

Here A is the value predicted by the Universal Soil Loss Equation and E_c the value derived from the Wind Erosion Equation. (All symbols are defined in Table 1.) Obviously, the variables in equation [1] must be comparable both in units and meaning for equation [1] to be valid.

Recent wind erosion research (Cole, 1984a, 1984b) has suggested that E_c represents a long-term statistical mean. Wischmeier and Smith (1978) have indicated that A is "the long-term average soil loss . . ." Consequently, assuming that the time and space intervals for both A and E_c are identical then A and E_c are additive and equation [1] is correct.

From equation [1] we further note that T is the upper limit of the sum of expected values of two random variables related to the process of soil erosion. In other words, soil erosion is a random process and hence its measures have statistical properties in addition to the mean, such as a variance and a probability density function.

It is the use of a simulated probability density function in conjunction with a lower limit and a probability statement that is proposed as a means of selecting acceptable soil erosion.

ANALYSIS

Soil erosion, w, has been defined (Cole, 1984b)* as

$$w \triangleq \frac{m}{A_p \tau} \qquad [2]$$

where m is the mass of soil lost from the projected surface A_p during time interval τ. When w is considered a random variable, then it is best described by its probability density function. The mean of w,

$$W \triangleq E(w) \qquad [3]$$

does not adequately constrain w. Consequently limiting w by T does not adequately limit w.
For example, if w were normally distributed with

$$W = T \qquad [4]$$

than 50% of the actual occurrences of w would be less than T. In other words, placing a limit on only the mean of soil erosion allows for a 50% chance of eroding more than T. This seems like a high risk situation.

This condition can be alleviated by placing restrictions on the distribution of w. We propose to limit the probability that w exceeds a specified value, w*, where the value of w* is chosen based on a productivity criterion similar to that of the T-value. At first this may seem identical to limiting W by T, however it should be noted that we are selecting a maximum value of w, *the random variable*, and not its *average*, W. Furthermore we are also specifying a probability of success, which is not done with the present use of T. For this method to be compatible with the concept of productivity implied in the T-value definition, two assumptions must be made.

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*In Cole (1984b), w represents the soil erosion due only to wind. Here we generalize it to represent both water and/or wind erosion in any combination.

That is, we can determine a minimum soil depth for which adequate productivity is sustained and we can select a minimum acceptable time at which this depth might occur.

The following analysis is facilitated by first discussing two of the assumptions implied by the use of W as the present soil erosion measure.

Assumptions

The generally accepted definition of T, i.e., ". . . the maximum level of soil erosion that will permit a *high level of crop productivity* to be sustained economically and *indefinitely*" (Wischmeier and Smith, 1978, as cited in the preface of Schmidt et al., 1982) embodies these assumptions.

First, if soil erosion as defined by equation [2] is to be *the* measure of the deleterious effects of erosion, then the effect on soil productivity *must* be linked via the equation of mass conservation. This equation, when applied to the soil erosion process, has been shown (Cole, 1985) to be

$$\frac{\partial}{\partial t}\int_{R(t)} \rho dR = \int_{S_1} g \cdot ds - \int_{S_2} f \cdot ds \quad \ldots\ldots\ldots\ldots [5]$$

The left-hand side of equation [5] represents the time rate of change of the mass of the soil within the ground, bounded by a control volume R(t) (Fig. 1) which is changing in time. The right-hand side represents the rates at which soil is formed at the bottom surface, S_1, and leaves at the top surface, S_2. It is these surface rates that cause the gain and loss of mass from the control volume. Here then we see the effect of the erosion process, as caused by the water and wind, on the soil mass within the control volume.

Since it is within the soil mass that the plant's roots reside, we see that the soil erosion and genesis rates cannot affect the instantaneous crop production rate if we neglect soil abrasion damage of the plant. It is the soil attributes of R, the volume, and ρ, the density, which can affect the crop production rate. As long as both R and ρ are adequate, the rates of erosion and genesis will not affect the crop production rate. Over a long period of time, the crop production rate could be affected if the soil loss rate exceeded the soil genesis rate. This unbalance could cause R and ρ to exceed their range of values that guarantee an adequate crop production rate. It is within this time interval, when crop productivity is adequate and independent of the erosion and genesis rates, that we choose to estimate a tolerable erosion rate from a knowledge of R and ρ.

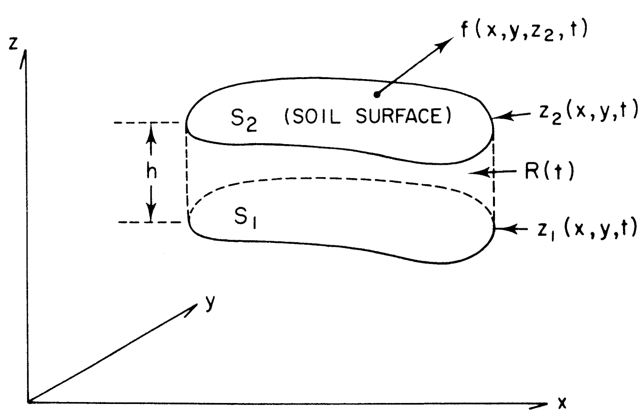

Fig. 1—The soil control volume and the coordinate system.

Obviously, the volume, R, of soil is affected by soil erosion, since the soil depth generally decreases. If all the soil were removed, then regardless of the values of ρ, productivity would be zero. However, some minimum depth is required to physically support the plant and to supply adequate moisture and nutrients. Consequently, one could visualize a minimum depth of soil, h*, assuming adequate ρ.

The situation with regard to a limit on p is not quite as clear. The soil mixture, which contains the nutrients, influences ρ. It is this chemical mixture that must be maintained within some range to guarantee adequate productivity. How the change in mixture proportions will affect density is not clear. In fact, it is conceivable that ρ would not change with time, while the mixture passed from an adequate to an inadequate state.

To analytically consider a mass mixture would require expanding equation [5] by having a mass continuity equation for each nutrient class and, consequently, an erosion and genesis rate for each class. Further complexity would be added when one realizes that the erosion process is generally selective, based on aggregate sizes that do not necessarily correlate to a nutrient class.

The second assumption associated with the T-value definition is that of the time duration during which crop productivity is to be sustained. The definition refers to this time interval as being indefinite which, if taken literally, would make the T-value definition ambiguous. We shall interpret indefinite to be defined for some time interval.

In order to see that this time interval is required to adequately limit W, we must integrate equation [5] with respect to time, divided by A_p and τ and then compute the statistical mean of w. The resulting equation is

$$G - W = \frac{1}{A_p} E \left\{ \frac{1}{\tau} \left[\int_{R(\tau)} \rho(x,y,z,\tau) dR - \int_{R(0)} \rho(x,y,z,0) dR \right] \right\}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [6]$$

where

$$W \triangleq E \left\{ \frac{1}{A_p \tau} \int_{\tau} \int_{S_2} f \cdot dsdt \right\} \quad \ldots\ldots\ldots\ldots [7]$$

$$G \triangleq E \left\{ \frac{1}{A_p \tau} \int_{\tau} \int_{S_1} g \cdot dsdt \right\} \quad \ldots\ldots\ldots\ldots [8]$$

W is still the average soil erosion as previously defined by equations [3] and [2]. Here W is described in terms of the time and space integrals of f, the soil flux vector. By analogy we have G for the average soil genesis.

We are now in a position to evaluate the effect of a defined "indefinite" time interval. If by indefinite one means for all time, then τ approaches infinity and we note from equation [6] that

$$W = G \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [9]$$

that is, the average soil erosion must equal the average soil genesis. This is only truly stable case, though supposedly impractical.

Equation [9] also can be obtained by making the final average depth equal to the initial average depth for a finite interval, i.e., no net soil loss.

Another possible relationship between W and G is

$$W < G \quad \ldots \quad [10]$$

which implies an increase in the soil depth. This also is generally not of importance, since the loss of soil is the dominant problem, which is expressed as

$$W > G \quad \ldots \quad [11]$$

For this situation we note that a time interval τ must be specified. In fact, for all cases except τ approaching infinity, the time interval is required to specify W (or w). This affect of τ also can be noted in equation [7], which is the definition of average soil erosion. (It should be carefully noted that τ, the accumulation or integration interval, is not generally one year).

In summary, we see that to sustain a high level of productivity indefinitely implies (from equations [1], [9] and [10])

$$W \leqslant T \leqslant G \quad \ldots \quad [12]$$

However, for a realistic situation, as expressed by equation [11], τ is not infinite. It must be specified! A further conclusion is that any specification other than equation [12] implies that eventually we will arrive at some level of productivity that is unacceptable and that to sustain productivity "indefinitely", we must adjust the average soil erosion (computed for all time greater than τ) to equal the average soil genesis.

We now see that specification of a T-value, when W > G, implies a range of acceptable values of W and eventual depletion of a crop production capacity unless other unknown factors intervene. The acceptance of W under these conditions should imply that τ represents the upper limit of time during which new concepts and technologies must be applied to overcome the loss of soil. Pierce et al., (1983), also refer to the need for specifying a time interval when determining, what they refer to as, a "T1" soil loss tolerance value, i.e., a T-value "based on perpetual productivity of soil". They stress the fact that the value of the time interval, referred to as a "planning horizon", is critical in determining the numerical value of the tolerance. We are in agreement with this, however based on the previous arguments there is no way by which depletion of the soil will allow "perpetual productivity".

Selection of τ requires consideration of many factors and, consequently, is quite difficult, yet in certain cases this has been done. For example, the concept of assigning T-values to ranges of rooting depth (McCormack et al., 1982, Appendix) when plotted with the constraint, T = W, results in a plot of soil depth versus time (McCormack et al., 1982). This plot is reproduced as Fig. 2. For an assumed minimum rooting depth of 25 cm, we note from Fig. 2 a τ of about 3000 years for the zero renewal rate condition.

Having established the need for specifying a depth and time, we must next consider the randomness associated with the erosion process, since this will eventually lead to the development of a probability statement.

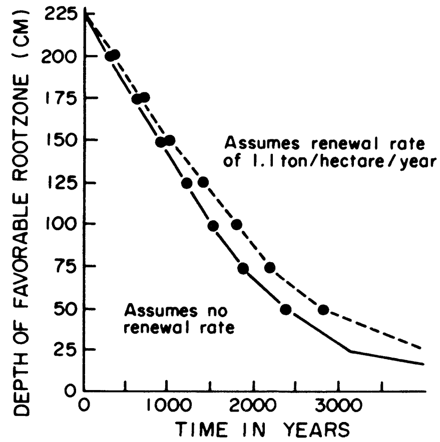

Fig. 2—Root zone depth vs. time for two renewal rates. (Adapted from McCormack et al., 1982, Fig. 1).

The Random Erosion Process

Development of the required probability statement is facilitated by first showing how the precursor of equation [6] appears in terms of random variables and then graphing this new equation for typical occurrences of the process.

In the development of equation [6], the last operation required to convert equation [5] to [6] was that of the statistical average. If we eliminate this step in the development, the following equation results:

$$\hat{g} - w = \frac{1}{A_p \tau} \left\{ \int_{R(\tau)} \rho(x,y,z,t)dR - \int_{R(0)} \rho(x,y,z,t)dR \right\} \quad \ldots \quad [13]$$

where $\hat{g}$ is the analog of w for the soil genesis at S_1. Equation [13] is quite general in that it allows for ρ to change in space and time. Development of a useful relationship for an acceptable soil erosion criteria requires certain assumptions.

First we assume that w will be evaluated for a region in which ρ is homogeneous in x and y, i.e.,

$$\rho = \rho(z,t) \quad \ldots \quad [14]$$

The reason for this assumption will be discussed later. Substitution of equation [14] into [13] and integrating over the area yields

$$\hat{g} - w = \frac{1}{\tau} \left\{ \int_{z_1(\tau)}^{z_2(\tau)} \rho(z,\tau)dz - \int_{z_1(0)}^{z_2(0)} \rho(z,0)dz \right\} \quad \ldots \quad [15]$$

Now we must modify the density function $\rho(z,\tau)$ since it is referenced to the z axis (defined in Fig. 1) where as data for a typical density function is referenced to the earths surface. See Fig. 3. This modification is expressed as

$$\rho(z,\tau) = \rho(z(\delta),\tau) = \hat{\rho}(\delta,\tau) \quad \ldots \quad [16]$$

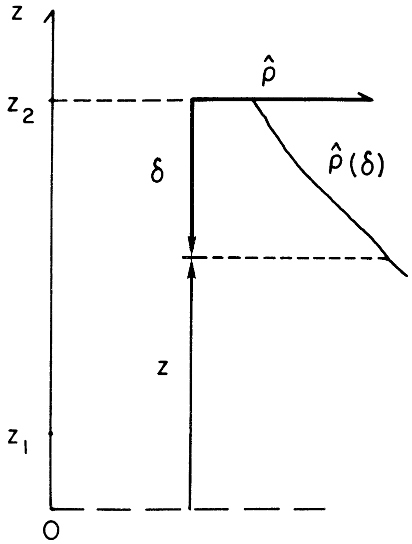

Fig. 3—The relationship between the z and δ axes.

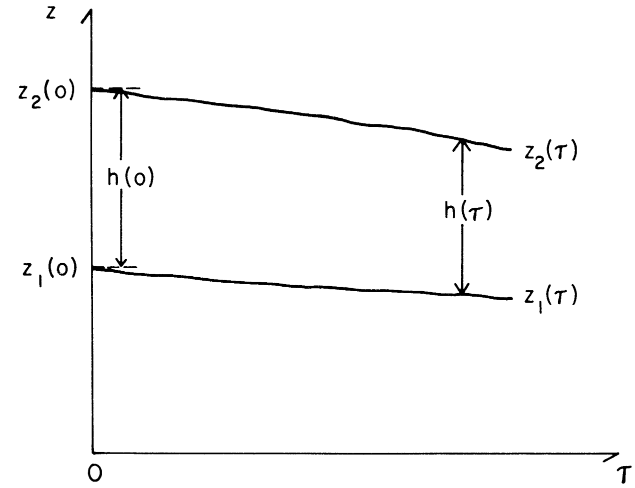

Fig. 4—The effective soil depth as a function of time and its relationship to z_1 and z_1 for all $z_2 > z_1 > 0$.

where

$$z + \delta = z_2(\tau) \qquad [17]$$

is the transformation between z and δ. Now substitution of equation [17] into equation [16] and the result into equation [15] yields

$$\hat{g} - w = \frac{1}{\tau}\left\{\int_{z_1(\tau)}^{z_2(\tau)} \hat{\rho}(z_2(\tau) - z,\tau)dz - \int_{z_1(0)}^{z_2(0)} \hat{\rho}(z_2(0) - z,0)dz\right\} \qquad [18]$$

Equation [18] shows that to determine w, for time interval τ, requires a knowledge of the density profile at the beginning and the ending of the time interval.

We can not integrate equation [18] because z_1 and z_2 are unknown, however, the difference is assumed known, i.e.,

$$h(\tau) \triangleq z_2(\tau) - z_1(\tau) \qquad [19]$$

where h is the effective soil depth. By returning to the definition of δ, i.e., equation [17] and utilizing equation [19] we can transform equation [18] into an integrable form.

$$\hat{g} - w = \frac{1}{\tau}\left\{\int_0^{\delta = h(\tau)} \hat{\rho}(\delta,\tau)d\delta - \int_0^{\delta = h(0)} \hat{\rho}(\delta,0)d\delta\right\} \qquad [20]$$

Fig. 4 depicts how the limits of integration of equation [20] shift with time.

Equation [20] can be further simplified by assuming that the only change in $\hat{\rho}$ with time is due to erosion at z_2 and genesis at z_1. Mixing of the soil layers between z_1 and z_2 is not considered. This assumption allows both integrals in equation [18] to become equal within the interval $(z_1(0), z_2(\tau))$. The result is a change in the ranges of the two integrals in equation [20], i.e.,

$$\hat{g} - w = \frac{1}{\tau}\int_{z_1(\tau)}^{z_1(0)} \hat{\rho}dz - \frac{1}{\tau}\int_{z_2(\tau)}^{z_2(0)} \hat{\rho}dz \qquad [21]$$

We note that the first integral in equation [21] is equal to $\hat{g}$ and the second to w.

Now one further simplification of equation [21] is appropriate if $w >> \hat{g}$. The result is

$$w = \frac{1}{\tau}\int_{z_2(\tau)}^{z_2(0)} \hat{\rho}(z_2(0) - z)dz \qquad [22]$$

which implies that if we integrate the original soil density profile over the depth of soil that has been eroded and divided by the time interval we arrive at a value of w. Equation [22] can be modified to accommodate the measurement variable h as follows.

Defining a new variable β as

$$\beta \triangleq z_2(0) - z; \qquad z \leqslant z_2(0) \qquad [23]$$

and transforming equation [22] accordingly we get

$$w = \frac{1}{\tau}\int_0^{z_2(0) - z_2(\tau)} \hat{\rho}(\beta)d\beta \qquad [24]$$

From Fig. 4, when $w >> \hat{g}$, then $z_1(0) = z_1(\tau)$, we note that

$$z_2(0) - z_2(\tau) = h(0) - h(\tau) \qquad [25]$$

Substituting this into equation [24] yields

$$w = \frac{1}{\tau}\int_0^{h(0) - h(\tau)} \hat{\rho}(\beta)d\beta \qquad [26]$$

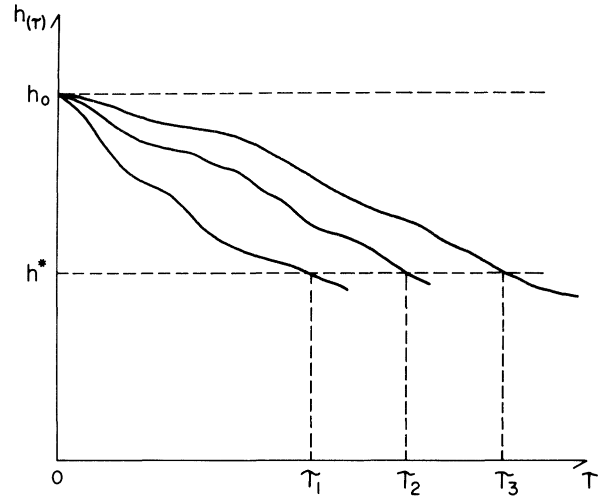

Fig. 5—Some possible realizations of h(τ).

To summarize, we see that the most general equation for w is equation [20]. Assuming that $\hat{\rho}$ does not change with time, e.g. no mixing, only "scalping" then equation [21] is applicable. If one assumes that $\hat{\rho}$ does not change in time and soil genesis is negligible, then equation [26] is useful. In the following we assume that equation [26] is appropriate. For those situations where it is not adequate, then either equation [20] or equation [21] may be used.

Equation [26] is a deterministic equation, developed using the conservation of mass principle, however, since w is random, it must be due to the randomness of either $h(\tau)$ or τ since $\hat{\rho}$ was considered to be time invariant. Now $h(\tau)$ will be random if either its functional relationship or τ is random.

Fig. 5 illustrates the randomness of h for three possible curves (or realizations) when $w >> \hat{g}$. Also shown is h^*, the minimum allowable value of h which will guarantee adequate productivity. The value of h^* would be determined from the productivity-depth curves similar to those shown in Timlin, et al., (1984) for the specific soil and crop.

Fig. 5 also shows three specific values of τ, which are referred to as τ_c the time of crossing, which is the time at which $h = h^*$.

Due to the randomness of the weather, there would be an infinity of these $h(\tau)$ curves and hence values of τ_c, which would result in a probability density function (pdf) for τ_c. For different h^*, we would except different pdf's, i.e., $p(\tau_c, h^*)$. Furthermore, different conservation and management practices would also change the pdf's!

We are now in a position to make a probability statement with regard to τ_c, provided we select the minimum desired τ_c, τ^*. We denote this probability as

$$\alpha_\tau \stackrel{\Delta}{=} P(\tau_c \geqslant \tau^*) \quad [27]$$

α_τ can be visualized as the probability of maintaining adequate crop productivity at least until τ^*. A typical pdf for τ_c $(p(\tau_c, h^*))$ is shown in Fig. 6, superimposed on a copy of Fig. 5, with the $h(\tau)$ suppressed for clarity. Also depicted is another pdf $(p(h(\tau^*)))$ which is the result of specifying a time τ^* and noting the values of $h(\tau^*)$ that randomly occur. The probability statement of interest here would be

$$\alpha_h \stackrel{\Delta}{=} P(h(\tau^*) \geqslant h^*) \quad [28]$$

which denotes the chances of a productive situation existing up to τ^*. This probability statement is identical to the first, hence we can conclude that

$$\alpha_\tau = \alpha_h \quad [29]$$

A heuristic proof of equation [29] can be had by observing in Fig. 6 that all sample functions of $h(\tau)$ that cross the line τ^* in the region below α_h also cross the line h^* only below the region α_τ. Therefore the frequencies of occurrence as represented by α_τ and α_h are equal. Todorovic and Woolhiser (1976) cite a proof of equation [29] when considering the random nature of local precipitation patterns. They refer to the process represented in Figs. 5 and 6 as the "first passage time" problem.

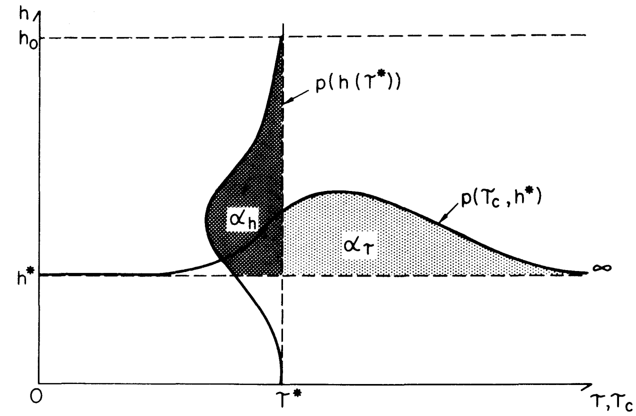

Fig. 6—Probability density functions of h(τ*) and τ_c.

As a consequence of equation [29], we can work with either τ_c or h as the random variable. We shall consider $h(\tau^*)$ as random. Note that

$$\alpha \stackrel{\Delta}{=} \alpha_h = \alpha_\tau \quad [30]$$

Determination and use of p(w)

In the previous section is was assumed that $p(h(\tau^*))$ was available, or could be simulated. It is more likely that any scheme for predicting soil erosion, w, will simulate $m(\tau)$ and compute w via equation [2]. However, if one simulated $h(\tau)$ it could be converted to w via equation [26] or one of its precursors depending on the assumptions made. Reasonable simulation methods for determining density functions related to soil erosion have been outlined by Rojiani et al., (1984) and Mills (1981).

Having determined $p(w(\tau^*))$, by multiple simulation runs, we must determine if this distribution is acceptable. An acceptable distribution would have the frequency of occurence of all $w \leqslant w^*$ greater than an acceptable amount α, i.e.,

$$P(w(\tau^*) \leqslant w^*) \geqslant \alpha \quad [31]$$

Where w^* is computed from equation [26] given h^* and τ^*. If equation [31] is not satisfied then other management strategies could be simulated or the value of α could be reduced.

We have now accomplished our initially stated goal. However, this scheme, while logically correct, is not practical due to the large simulation times implied by τ^*. For example, from Fig. 2, τ^* could be in the order of 1000 to 2000 years which far exceeds the antipicated simulation capability of 20 to 30 years. Furthermore predictions extending out that far are probably questionable based on our knowledge of future weather and management techniques.

A reasonable approach to determining a w* for some short τ would be to partition the depth of soil to be eroded, $(h^* - h(0))$, linearly with time. This would be equivalent to "using up" the productive capacity of the soil equally with time. The short term limit for prediction, h^+, for $\tau < \tau^*$, would be

$$h^+ = \frac{(h^* - h(0))}{\tau^*} \tau^+ + h(0) \quad \ldots\ldots\ldots\ldots\ldots [32]$$

If $\hat{\rho}$ were uniform with depth, when w^+, the limit based on h^+ would be equal to w*, however, this is generally not to be expected.

A probability statement analogous to equation [31] for testing the adequacy of the short term distribution $p(w(\tau^+))$ is

$$P(w(\tau^+) \leqslant w^+) \geqslant \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [33]$$

While equation [33] is more restrictive than is necessary to meet the final criterion of equation [31] it is sufficient. One advantage of the subdivision of τ^* into a series of say 30 year subintervals, allows making "course corrections" in our "navigation" through the h,τ space implied in Figs. 3 and 4. These corrections involve recomputing w* to allow for any gains or losses relative to w^+ which have accrued during the previous time interval.

Of course criteria other than equation [32] are possible but they must satisfy equation [31].

One further point must be made. Just prior to equation [14] it was assumed that ρ was not a function of x nor y. This assumption was required because of the difficulty of assigning a single productivity rate to non homogeneous areas. For example, if one has two different density profiles within a field, then for a single field productivity rate we would have many two value sets for h*. Some sets could conceivably reduce one area of the field to a totally unproductive state. This problem is avoided by the previous assumption, where each area is assigned its limit. Obviously this is an important problem dealing with resource allocation on a larger scale than a field.

CONCLUSIONS

1. Specification of an upper bound on the mean soil erosion does not adequately limit the soil erosion process. For example, if the process is normally distributed, then half of the actual occurrence could exceed the specified soil loss tolerance.

2. The selection of acceptable soil erosion values, when soil erosion exceeds soil genesis, requires that an acceptable risk be specified along with a minimum depth and time. The depth is the minimum required to guarantee adequate crop productivity. The time is the desired minimum time interval before reaching this depth.

3. The adequacy of short term erosion predictions must be based on a short term erosion limit w^+ which is related to *the* erosion limit w*.

References

1. Cole, G. W. 1984a. A method for determining field wind erosion rates from wind-tunnel-derived functions. TRANSACTIONS of the ASAE 27(1):110-116.
2. Cole, G. W. 1984b. A stochastic formulation of soil erosion caused by wind. TRANSACTIONS of the ASAE 27(5):1405-1410.
3. Cole, G. W. 1985. Some wind erosion process measures. TRANSACTIONS of the ASAE 28(4):1108-1114, 1123.
4. McCormack, D. E., K. K. Young, and L. W. Kimberlin. 1982. Current criteria for determining soil loss tolerance. Chapter 9 in Schmidt, B. L., R. A. Allmaras, J. V. Mannering, and R. I. Papendick, editors, Determinants of Soil Loss Tolerance, ASA Special Publication No. 45, Am. Soc. Agr., Madison, WI 53711.
5. Mills, W. C. 1981. Deriving sediment yield probabilities for evaluating conservation practices. TRANSACTIONS of the ASAE 24(5):1199-1203, 1210.
6. Pierce, F. J., W. E. Larson, R. H. Dowdy, and W. A. P. Graham. 1983. Productivity of soils: Assessing long-term changes due to erosion. J. Soil and Water Conservation 38(1):39-44.
7. Rojiani, K. B., K. A. Tarbell, V. O. Shanholtz, and F. E. Woeste. 1984. Probabilistic modeling of soil loss from surface mined regions. TRANSACTIONS of the ASAE 27(6):1798-1804.

(continued on page 1932)

TABLE 1. NOTATION. (M, L, AND T AS DIMENSIONS REFER TO MASS, LENGTH, AND TIME)

Symbol	Definition and dimensions
A	the long time average soil loss predicted from the universal soil loss equation, $M\ L^{-2}T^{-1}$
A_p	the projected area of the field surface on the x,y plane, L^2
$E(\cdot)$	statistical mean, dimensions vary
E_c	potential average annual soil loss as defined in Woodruff and Siddoway (1965), $M\ L^{-2}T^{-1}$
f	soil flux vector, $M\ L^{-2}T^{-1}$
g	soil genesis flux, $M\ L^{-2}T^{-1}$
$\hat{g}$	the time and space average of the normal component of g, soil genesis, $M\ L^{-2}T^{-1}$
G	expected value of $\hat{g}$, $M\ L^{-2}T^{-1}$
h	effective soil depth, (Sanders, 1982 pg 43) and, equivalently, the thickness of the control volume, see Fig. 1, L
h_0	the initial value of h, L
h*	the minimum value of h that guarantees an acceptable productivity, L
h+	see equation [32], L
m	soil loss, the mass of soil lost from surface S_2 during the time interval τ, M
p	a probability density function, dimensions vary
$P(\cdot)$	probability statement, dimensionless
R	the control volume, L^3
S	surface area of R, the control volume, L^2
S_i	surface area of the i-th surface of the control volume, L^2
t	time, T
T	soil loss tolerance, $M\ L^{-2}T^{-1}$
w	the time and space average of the normal component of the surface soil flux vector, i.e., soil erosion, $M\ L^{-2}T^{-1}$
w*	the maximum desired $w(\tau^*)$, $M\ L^{-2}T^{-1}$
w+	the maximum desired $w(\tau+)$, $M\ L^{-2}T^{-1}$
W	the expected value of w, e.i., average soil erosion, $M\ L^{-2}T^{-1}$
x	distance along the x axis, L
y	distance along the y axis, L
z	distance along the z axis, L
α	a probability - see equation [30], dimensionless
α_h	a probability - see equation [28] and Fig. 6, dimensionless
α_τ	a probability - see equation [27] and Fig. 6, dimensionless
β	a dummy variable for integration, see equation [23], L
δ	the vertical coordinate referenced to z_2, see equation [17] and Fig. 3, L
ρ	soil density function referenced to the coordinate system, $M\ L^{-3}$
$\hat{\rho}$	soil density function referenced to z_2, $M\ L^{-3}$
τ	a time interval, T
τ_c	the time at which h = h*, T
τ_i	the time at which h crosses the line h = h*, T
τ^*	the minimum desired τ_c, see equation [27], T
$\tau+$	the minimum duration for simulation of w, T
Subscripts	
i	index 1, 2, 3 . . . various surfaces, times, or intervals
Other symbols	
$\triangleq$	defined

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(continued from page 1926)

8. Sanders, J., editor. 1983. Resource Conservation Glossary. Soil Conservation Society of America, Ankeny, IA 50021.

9. Schmidt, B. L., R. R. Allmaras, J. V. Mannering, and R. I. Papendick, editors. 1982. Determinants of Soil Loss Tolerance, ASA Special Publication No. 45, Am. Soc. Agr., Madison, WI 53711.

10. Skidmore, E. L., N. P. Woodruff. 1968. Wind erosion forces in the United States and their use in predicting soil loss. USDA-ARS, Agricultural Handbook No. 346, 42 pp.

11. Timlin, D. J., R. B. Bryant, V. A. Snyder, and R. J. Wagenet. 1984. Modeilng corn yields and the decline in soil productivity in relation to soil erosion in New York state. Poster Session No. 8, National Symposium on Erosion and Soil Productivity. December 10, 11, 1984. New Orleans, LA.

12. Todorovic, P., and D. Woolhiser. 1976. Stochastic structure of the local pattern of precipitation. Chpt. 15 in Stochastic Approaches to Water Resources, Vol. 2, H. W. Shen editor and publisher, P.O. Box 606, Ft. Collins, CO 80521 USA.

13. Wischmeier, W. H., and D. D. Smith. 1978. Predicting rainfall erosion losses - a guide to conservation planning. USDA, Agr. Handbk. No. 537.

14. Woodruff, N. P., and F. H. Siddoway. 1965. A wind erosion equation. Soil Sci. Soc. Am. Proc. 29(5):602-608.